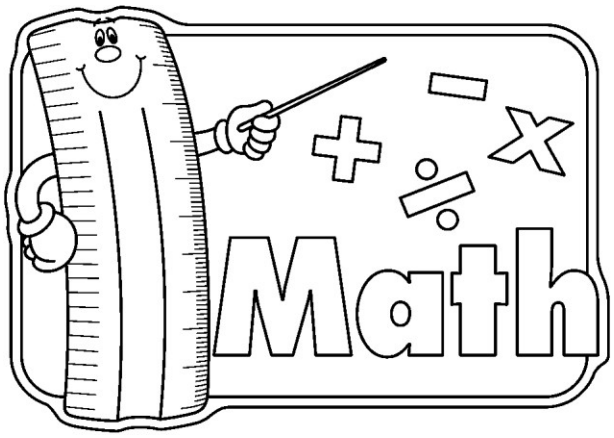
Barrington Public Schools
Elementary Math
2013-2014

Math Models and Terminology
Grades K-5



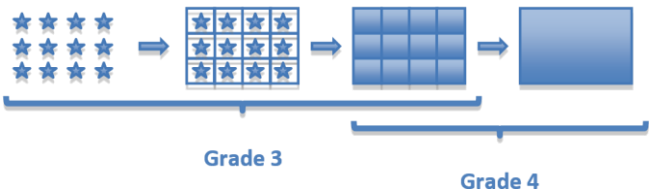
(Adapted from "How to Implement Story of Units", Eureka Math)

Math Models

Math models are consistently utilized as part of our instruction and learning activities to promote student understanding of math content and skills. A finite set of concrete and pictorial models are used throughout the grades. Students build increasing dexterity with these models through persistent use within and across levels of curriculum. The repeated appearance of familiar models helps to build critical vertical links between topics of one grade level and the next.

A description of each of the models follows:

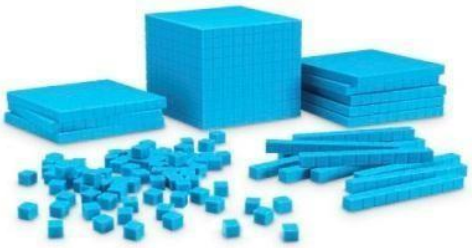
Array and Area Model (Grades 1-5)



An array is an arrangement of a set of objects organized into equal groups in rows and columns. Arrays help make counting easy. Counting by equal groups is more efficient than counting objects one by one. The ten-frame is an array used in Kindergarten. Students count objects in arrays in Kindergarten. The rectangular array is used to teach multiplication and leads to understanding area in grade 3.

Arrays reinforce the meaning of multiplication as repeated addition (e.g., 3 x 4 = 4 + 4 + 4), and the two meanings of division—that 12 ÷ 3 can indicate how many will be in each group if I make 3 equal groups and that it can also indicate how many groups I can make if I put 3 in each group. Further use of arrays in later grades reinforces the relationship between multiplication and division.

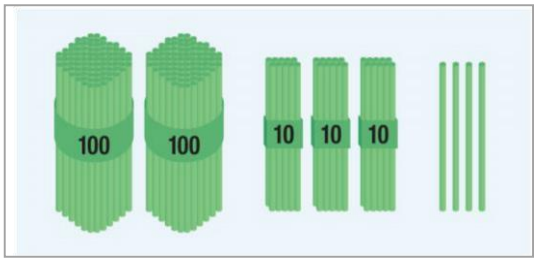
Base 10 Blocks (Grades K-2)



Base-ten blocks include thousands "cubes," hundreds "flats," tens "rods," and ones. Base-ten blocks are a proportional representation of units of ones, tens, hundreds, and thousands and are useful for developing place value understanding.

Base-ten blocks are introduced after students have learned the value of hundreds, tens, and ones and have had repeated experiences with composing (putting together) and decomposing (taking apart) groups of 10 ones or groups of 10 tens with bundles.

Bundles (Grades K-2)

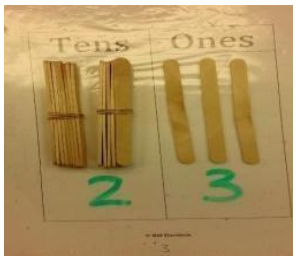


Bundles are discrete groupings of place value units (tens, hundreds, thousands) usually made by students/teachers placing a rubber band around straws, popsicle sticks, or coffee stirrers. Linking cubes may also be used in this fashion. Ten straws (or cubes) are bundled (or linked) into 1 unit of ten, 10 tens are bundled into 1 unit of a hundred, and so on. These student-made groupings provide the necessary conceptual foundation for children to be successful with pre-grouped, proportional, and nonproportional base-ten materials. (See Base-Ten Blocks and Number Disks.)

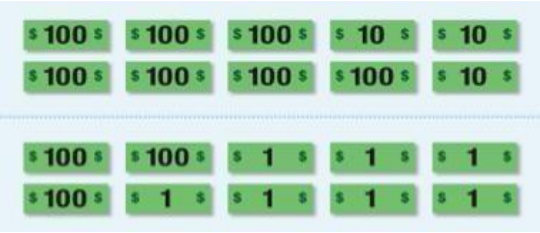
Understanding tens and ones is supported in Kindergarten as students learn to compose (put together) and decompose (take apart) tens and ones by "bundling" and "unbundling" the materials. Numbers 11-19 are soon seen as 1 ten (a bundled set of 10 ones) and some extra ones.

By Grade 2, students expand their skill with and understanding of units by bundling units of ones, tens, and hundreds up to one thousand with sticks. These larger units are discrete and can be counted: "1 hundred, 2 hundred, 3 hundred, etc." Bundles also help students extend their understanding of place value to 1000 in grade 2. Repeated bundling experiences help students to internalize the pattern that 10 of one unit make 1 of the next larger unit. Expanded form, increased understanding of skip-counting in Grade 2 and fluency in counting larger numbers are all supported by the use of this model.

Bundles are also useful in developing conceptual understanding of renaming in addition and subtraction. The mat below shows 2 tens and 3 ones. To solve 23 – 9, one bundle of ten is "unbundled" to get 1 ten and 13 ones in order to take away 9 ones.



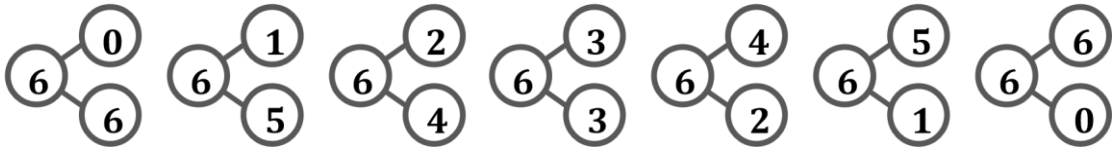
Money (Grade 2)



Dollar bills (1s, 10s, and 100s) are non-proportional units that are used to develop place value understanding. That is, bills are an abstract representation of place value because their value is not proportionate to their size. Ten bills can have a value of $10 or $1000 but appear identical aside from their printed labels. Bills can be "traded" (e.g., 10 ten-dollar bills for 1 hundred-dollar bill) to help students learn equivalence of the two amounts.

As with other place value models, students can use bills to model numbers up to three digits, to read numbers formed with the bills, and to increase fluency in skip-counting by tens and hundreds.

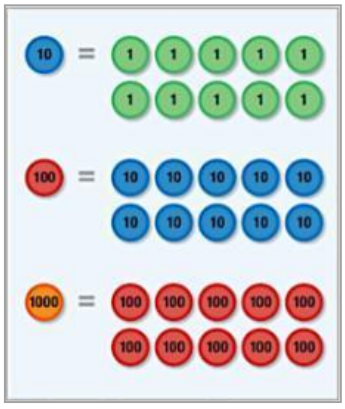
Number Bond (Grades K-5)

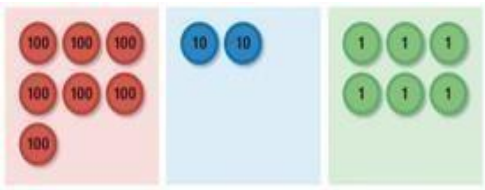


The number bond is a pictorial representation of part-part-whole relationships and shows that within a part whole relationship, smaller numbers (the parts) make up larger numbers (the whole). The number bond may be presented as shown, using smaller circles (or squares) for the parts to distinguish the part from the whole. As students become more comfortable using number bonds, they may be presented using the same size shape for parts and whole.

Number bonds of 10 have the greatest priority because students will use them for adding and subtracting across 10. Students move towards fluency in Grade 1 with numbers to 10 building on the foundation laid in Kindergarten. They learn to decompose (take apart) numbers to ten with increasing fluency in Grade 1. Students learn the meaning of addition as "putting together" to find the whole or total and subtraction as "taking away" to find a part.

Number Discs (Grades 2-5)





Place Value Chart with Number Disks

Number disks are non-proportional units used to further develop place value understanding. Like money, the value of the disk is determined by the value printed on it, not by its size. Number disks are used by students through Grade 5 when modeling algorithms (step-by-step procedure for calculation) and as a support for mental math with very large whole numbers. Whole number place value relationships modeled with the disks are easily generalized to decimal numbers and operations with decimals.

Number Line (Grades K-5)

The number line is used to develop a deeper understanding of whole number units, fraction units, measurement units, decimals, and negative numbers. Throughout Grades K-5, the number line models measuring units.

Number Path (Grades K-1)

1	2	3	4	5	6	7	8	9	10

The number path can be thought of as a visual (pictorial) representation of the number tower (see description below) and is foundational to understanding and using the number line. It also serves as a visual representation of 1:1 correspondence and the concept of whole numbers (one number, one space, and each being equal in size). The color change at 5 helps to reinforce the 5 and 10 benchmarks. The number path also serves as an early precursor to measurement concepts and a support for cardinal counting. (If a student places 7 objects in each of the 7 spaces on the path, they must realize that there are 7 objects, not 10. Simply because the path goes up to 10 does not mean there are 10 objects.)

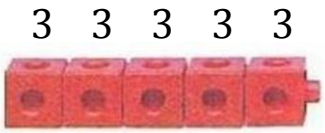
Number Towers (K-3)

Number towers, also known as number stairs, are representations of quantity constructed by joining together interlocking cubes. In the early grades, they are used to help younger children quite literally build their knowledge of cardinality by erecting towers of various numbers. Number towers are then used to teach concepts of "more/less" globally and the patterns of "1 more/less" and "2 more/less" specifically. This model leads to an understanding of comparison and the word "than," not only in the context of "more than" and "less than," but also in the context of "taller than," "shorter than," heavier than," "longer than," etc.

Children are encouraged to build towers for quantities 1 through 5 in one color. Quantities beyond 5 are added on in a second color. This color change provides support for several important developmental milestones. First, it facilitates children's understanding of 5 as a benchmark, which provides an important beginning to their ability to subitize (instantly seeing "how many). Second, it allows students to see relationships such as "5 needs 2 more to be 7;" "5 is 1 less than 6;" and "5 and 4 is 9, which is 1 less than 10." Finally, it encourages students to count on from 5 rather than starting at 1 to count quantities of 6, 7, 8, 9, and 10.

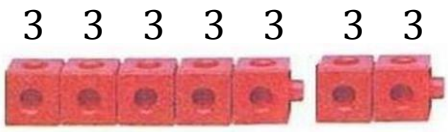
Such comparisons lead to looking at the parts that make up a number. ("3 is less than 7. 3 and 4 make 7.") These concepts are foundational to students' understanding of part/whole models (see Number Bonds). This, in turn, leads naturally to discussions of addition and subtraction, fact fluencies (+1, +2, +3, -1, -2, -3), and even the commutative property (flip the tower; 3 + 4 or 4 + 3—does the whole change?), which are explored in Kindergarten and Grade 1.

In Grades 2 and 3, as students prepare for and study multiplication and division, each unit in the number stair can be ascribed a value other than 1. For example: "Each of our cubes is equal to three. What is the value of the stair with five cubes?"



Further, the use of number stairs can be extended to help children understand more complex properties like the distributive property. "Each of our cubes is equal to three. Make a stair with five cubes. Now add two more cubes. The stair with 7 cubes is 2 more threes. So, 5 threes is 15, 2 threes is 6, and together 7 threes is 15 + 6 or 21."

5 threes + 2 threes = (5 + 2) threes



Place Value Chart (Grades 2-5)

The place value chart is a graphic organizer that students can use (beginning in Grade 1 with tens and ones through Grade 5 with decimals) to see the coherence of place value and operations between different units.

Place Value Chart Without Headings

(Used with labeled materials such as disks)

Place Value Chart with Headings

(Used with unlabeled materials such as base-ten blocks or bundles)

Hundreds	Tens	Ones

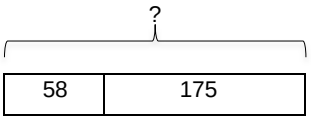
Rekenrek (Grades K-5)

20-Bead Rekenrek

100-Bead Rekenrek

The Rekenrek has a 5 and 10 structure, with a color change at 5 (eliciting the visual effect of grouping 5 and grouping 10). The 20-bead Rekenrek consists of 2 rows of 10 beads, allowing students to see numbers to 10 either as a number line on one row or a ten-frame (5 beads on two rows). A 100-bead Rekenrek has 10 rows of 10 beads. Other names for the Rekenrek are "Calculating Frame," "Slavonic Abacus," "Arithmetic Rack," or "Math Rack."

Tape Diagram or Bar Diagram (Grades 1-5)



Rachel collected 58 seashells. Sam gave her 175 more. How many seashells did she have then?

Tape diagrams, also called bar models, are pictorial representations of relationships between quantities used to solve word problems. Students begin using tape diagrams in 1st grade, modeling simple word problems involving the operations. It is common for students in 3rd grade to express that they don't need the tape diagram to solve the problem. However, in Grades 4 and 5, students begin to appreciate the tape diagram as it enables students to solve increasingly more complex problems.

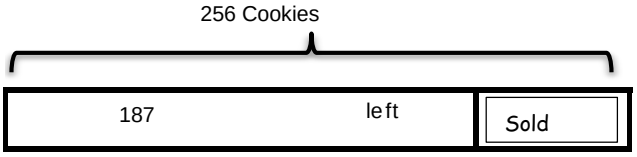
At the heart of a tape diagram is the idea of forming units. In fact, forming units to solve word problems is one of the most powerful examples of the unit theme and is particularly helpful for understanding fraction arithmetic.

The tape diagram provides an essential bridge to algebra and is often called "pictorial algebra."

There are two basic forms of the tape diagram model. The first form is sometimes called the part-whole model; it uses bar segments placed end-toend (Grade 3 Example below depicts this model), while the second form, sometimes called the comparison model, uses two or more bars stacked in rows that are typically left justified. (Grade 5 Example below depicts this model.)

Grade 3 Example

Sarah baked 256 cookies. She sold some of them. 187 were left. How many did she sell?

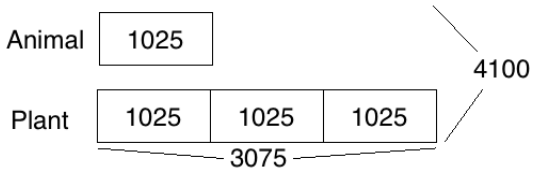


256 – 187 = ?

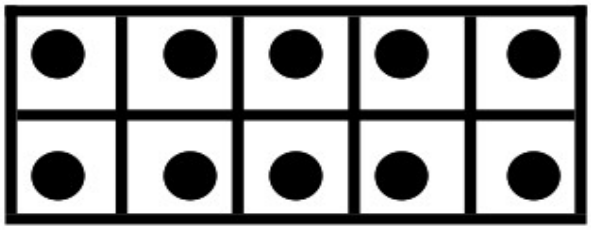
Sarah sold ? cookies.

Grade 5 Example

Sam has 1025 animal stickers. He has 3 times as many plant stickers as animal stickers. How many plant stickers does Sam have? How many stickers does Sam have altogether?



Ten-Frame (Grades K-3)



A ten-frame is a 2 by 5 grid (array) used to develop an understanding of concepts such as 5-patterns, combinations to 10, and adding and subtracting within 20. The frame is filled beginning on the top row, left to right, then proceeding to the bottom row building left to right. This pattern of filling supports subitizing (instantly seeing "how many") by building on the 5 benchmark, as well as providing a pattern for placing disks on place value mats in later grades. Concrete counters as well as pictorial dots may be used to represent quantities on the frame.

In Kindergarten and in early Grade 1, a double ten-frame can be used to establish early foundations of place value (e.g., 13 is 10 and 3 or 1 ten and 3 ones) and can also be used on place value mats to support learning to add double digit numbers with regrouping. The "completion of a unit" on the tenframe in early grades empowers students in later grades to understand a "make 100 (or 1000)" strategy, to add 298 and 37 (i.e., 298 + 2 + 35), and to more fully understand addition and subtraction of measurements (e.g., 4 ft. 8 in. + 5 in).